Modern Physics Letters B

NANOSCALE MAGNETIC TRAPS

I. F. LYUKSYUTOV
Physics Department, Texas A&M University,
College Station TX 77843-4242, USA

Received (to be inserted
Revised by Publisher)

We show that nanofabricated magnetic textures allow the trapping and manipulation of nanosize diamagnetic systems, such as carbon nanotubes, proteins and membranes as well as cold atoms. The latter can have temperatures as high as 1K. Magnetic textures, which can be used as traps, include films, dots and nanowires, both single and in arrays. Manipulation with trapped nanoparticles/atoms is possible by using external magnetic fields. We also briefly discuss prospects for magnetic traps at the micron scale.

In 1839 Earnshaw[1] proved that no stable equilibrium is possible if particles interact according to Coulomb's law, or any interaction having $1/R^2$ behavior. Later Lord Kelvin[2] showed that diamagnetic materials represent an exception: for them, stable equilibrium is possible. In 1939 Braunbek[3] published a detailed study of diamagnetic levitation. He experimentally observed the levitation of graphite and bismuth. In the Braunbek experiment the maximum field variation was about 2.4T over the distance of 2mm. Recently experimental studies of levitation have been performed by Geim and coworkers[4–6]. A brief review on diamagnetic levitation is given in Ref.6. Magnetic fields of about 0.005 T generated by currents in microwires 50-100μm wide on semiconductor chips have been recently used to create magnetic traps to capture ultra-cold atoms (which have a magnetic moment) several hundred microns above the surface of the chips (the so-called Atomic Chip). Using these traps, experimentalists manipulated ensembles of atoms[7,8], and observed Bose-Einstein condensation[9]. Different types of current based planar magnetic traps have been discussed, e.g. in Ref.10. In this article I show that traps can be realized as magnetic nanostructures. Although it is easier to control the magnetic field by varying the current, the use of magnetic nanostructures as traps has important advantages. Indeed, the stray magnetic field at the surface of a magnetic film, magnetic nanowire or magnetic dot can be of the order of $2\pi M$, where $\vec{M}$ is the magnetization. This corresponds to the Tesla range. A second advantage is that magnetic elements of nanoscale size are in a single domain state. As a result, the magnetic field goes outside of the magnetic structure in the contrast to the multido-

main state for which the magnetic flux may be closed inside the magnet. Thus a magnetic field of the order of a Tesla may appear on the surface of magnetic nanostructures. As a result, nanostructure based magnetic traps can produce a strong magnetic field in the range of a Tesla, and huge gradients of the magnetic field, up to 10^8T/m. This value of magnetic field gradient is 10^6 times larger than that used in Atomic Chips.

The dramatic increase in field gradient, i.e. in the ponderomotive force, opens a new direction in application of magnetic traps: trapping of diamagnetic objects. In this case, the important parameter is $G_E = (1/2)|\nabla B^2|$ which is proportional to the magnitude of the force density which acts on a diamagnetic substance. In the case of the Braunbek experiments[3] the maximum value of G_E was about $2.9\times 10^3 \mathrm{T}^2/\mathrm{m}$. According to Ref.[6], the threshold value for water levitation is 1400 T^2/m. As proposed in this article, it is feasible to achieve values of $G_E \sim 10^8 \mathrm{T}^2/\mathrm{m}$. At such values of G_E, the magnetic force is much greater than the gravitational force which can be ignored. The general problem of magnetic traps for nanoparticles goes far beyond the scope of this letter. Below we only formulate some simple generic models and describe briefly their properties.

Linear Magnetic Trap: Consider a diamagnetic rod (e.g. nanotube, or protein molecule) of volume V, length L and magnetic susceptibility $\chi < 0$. Let the magnetic field be generated by the edge of a magnetic film (covering half of an infinite substrate) with the thickness d and magnetization M, normal to the film. At a distance R much larger than d the magnetic film edge generates a field of strength $2Md/R$. Let an external magnetic field B_0 be applied normal to the film and antiparallel to the stray magnetic field above the film edge. Then the net magnetic field on a straight line parallel to the film edge at the distance $R_0 = d(2M/B_0)$ is zero. As a result, stable trapping is possible. The energy per unit length E_t of a diamagnetic rod placed parallel to the edge at a distance R from the magnetic layer is given by:

$$E_t = -\frac{\mu - 1}{\mu + 1}\frac{V}{\pi L}B^2(R). \qquad (1)$$

The value $1-\mu$ is 1.6×10^{-4} for graphite, 1.7×10^{-4} for bismuth and 8.8×10^{-6} for water; B is the total magnetic field. The stable equilibrium point is at the distance of $R_0 = d(2M/B_0)$ above the edge of the film. This type of magnetic field distribution has been used to trap ultra-cold atoms above atomic chips[7,8]. The linear trap allows particles to move along the trap. It can be closed by changing the direction of the magnetic film edge (e.g., rotate it by $\pi/2$ similar to the traps used in atomic chips[7]).

Although magnetic field gradients are large, the depth of the potential well for a small diamagnetic nanoparticle is small since the diamagnetism is normally weak. The potential well can be made deeper by using large nanoparticles like nanotubes, proteins and membranes. Indeed, in a field of 1T the induced magnetic moment for 10^4 carbon atoms ($|\chi| \sim 10^{-4}$) is of the order of one μ_B which corresponds to a potential well depth of the order of 1K. Such temperatures can be achieved by using helium vapor cooling. By increasing the nanotube (molecule) size by a factor of 10^3, it could be possible to achieve stable trapping at room temperatures.

The above analysis is valid for the case of a film with in-plane magnetization $\vec{M}$, perpendicular to the edge. The magnitude of the field is $2\pi Md/R$ and the field is directed radially toward or from the edge. The magnetic trap in this case can be created by an applied magnetic field normal to the film and antiparallel to the stray field from the film edge.

A magnetic dot of thickness d and radius r_d with magnetization M normal to the film produces a stray field which is similar to the magnetic field from a current loop with $I = cMd$ along the edge of the dot. The magnetic field along a straight line which is normal to the dot and passes through its center, is normal to the film and is equal to $B_z = (2\pi Mdr_d^2)/(r_d^2 + R^2)^{3/2}$, where R is the distance from the dot. With uniform externally applied magnetic field opposite to B_z, it is possible to obtain zero field at some point above the dot. A simple analysis shows that the vicinity of this point represents a stable trap for a diamagnetic nanoparticle. The position of the trap can be easily shifted by variation of the external field.

Magnetic Field from an Array of Magnetic Nanorods: A convenient approach to preparation of nanostructured materials is to synthesize the desired material within the pores of a porous membrane (template)[11]. Asymmetric diblock copolymer films[12], molecular sieves, track-etched mica[13], polymer membranes[14] and porous anodized alumina membranes[15] are some representative examples for such templates. Porous alumina membranes are prepared electrochemically from aluminum metal[16]. Under specific conditions of anodization, it is possible to obtain a highly ordered hexagonal array of pores, with pore densities as high as 10^{11} pores/cm^2 (Ref.[15]). Such alumina membranes with pore diameters down to 10 nm are commercially available. General magnetic characteristics (e.g. saturation magnetization, coercive field, etc.) of arrays of nanorods (nanowires) fabricated in alumina templates were studied by many groups (see review in Ref.[15]). However, we are not aware of any study of the magnetic properties at the micrometer or nanometer scale. Below we discuss magnetic properties at such scales in connection with magnetic nanotraps.

An infinitely long magnetic rod produces no external magnetic field. The magnetic flux is concentrated inside the infinite rod. If the Magnetic Nanorod (MNR)is finite, the magnetic flux goes outside. Consider a periodic square array of magnetic rods with lattice constant a. Let the length L and radius $R \sim a$ of each rod be such that the ratio $L/R = \mathcal{N} \gg 1$. The value of $\mathcal{N}$ can be as large as 10^2. The lateral scale of the alumina template can be in the *cm* range. It is easy to evaluate the magnetic field inside the template with MNR's using a similarity with the electric field from a capacitor. The stray magnetic field from MNR's with parallel magnetization produces a homogeneous field opposite to the MNR magnetization direction inside the template. The magnetic field is inhomogeneous near the MNR ends on a length scale of the order of a. This deviation from homogeneity disappears exponentially with distance from the surface. The magnetic field values can be found from the condition of zero net flux in the system (the net flux produced by each rod is exactly zero). If the unit cell area of the MNR lattice is A, then the magnetic field $\vec{H}_i$ *inside* the template, but *outside* the MNR is directed opposite to $\vec{M}$ and is given by: $\vec{H}_i = -4\pi\vec{M}(\pi R^2/A)$. Inside the MNR $\vec{B} = 4\pi\vec{M} + \vec{H} = 4\pi\vec{M}(1 - (\pi R^2/A))$.

In this particular example we assume that the coercive field is larger than $|H_i|$. This assumption is not restrictive in a real experiment: an external magnetic field parallel to the rod magnetization direction will keep the direction of magnetization. Consider two square sublattices on the template surface, one with lattice sites in the centers of MNR ends and the other its conjugate. The magnetic field on these two sublattices is directed normal to the template and has the opposite direction on each sublattice. The in-plane magnetic field component is zero due to the symmetry. Consider the field on the template surface outside the MNR. The magnetic field can be found by using the electrostatic analogy and solving Laplace's equation with the surface charge density $div\vec{M}$. Such "charges" on the MNR on one end can create only an in-plane field component on the surface of the template at that end. This means that the normal component of the field is due to the MNR "charges" on the other side of the template: $H_z^{(s)} = -2\pi\vec{M}(\pi R^2/A)$. On the surface of the MNR the normal component of the magnetic field is given by: $H_z^{(MNR)} = 2\pi\vec{M}(1-(\pi R^2/A))$. This consideration assumes that the magnetization $\vec{M}$ is directed strictly along the MNR. In a real situation, the magnetic field distribution can deviate from the idealized picture above. The MNR array could produce very large magnetic field gradients, up to 10^8T/m. Similar arguments can be used for an array of magnetic dots with magnetization normal to the dot and to the magnetic film with normal magnetization and with an array of holes (anti-dots). Both arrays of nanowires and dots/anti-dots can be used for trapping single diamagnetic particles, extended objects like membranes or cold atoms.

Multilayered Magnetic Trap: The large gradients of the magnetic field can be achieved with a layered structure built from alternating magnetic and non-magnetic layers. This structure should have well defined edge. For simplicity we assume that the plane of the edge is normal to the film plane. Consider the case when the magnetization $\vec{M}$ is parallel to the multilayered film plane and is normal to the edge surface. To analyze the magnetic field distribution, we can apply the same approach as in the case of the array of magnetic nanorods. If the period of the layered structure is D and magnetic layer thickness is D_m, then the magnetic field $\vec{H}_i$ *inside* the non-magnetic layer, is given by: $\vec{H}_i = -4\pi\vec{M}(D_m/D)$. Inside magnetic layer $\vec{B} = 4\pi\vec{M} + \vec{H} = 4\pi\vec{M}(1-(D_m/D))$. Consider two sets of lines parallel to the layers on the film edge with one set in the centers of the surfaces of the magnetic and and the second set in the centers of the non-magnetic layers. The magnetic field on these two sublattices is directed normal to the edge surface and has the opposite direction on each sublattice. The in-plane magnetic field component is zero due to the symmetry. The normal component of the magnetic field on the surface of the magnetic layer is given by: $H_z^{(s)} = 2\pi M(1-(D_m/D))$. As in the case of the MNR array the layered structure can generate magnetic field gradients as large as 10^8T/m. The multilayer geometry can be realized at nanoscale by growing multilayer films. A well known example is that of sensors based on the Giant Magnetoresistance (GMR) effect. Though it is feasible to grow a stack of such layered structures with edges, we are not aware of any particular experimental realization. The thickness of such structures can be limited at the nanoscale e.g. due to the mismatch of the

lattice periods of magnetic and non-magnetic films. However, this type of structure can be easily realized at the submillimeter or micron scale (see below).

Array of Linear Magnetic Traps: As a simple model of the array of the linear magnetic traps, we propose to use a magnetization distribution in the form $M = M_0 \sin(kx)$, where $k = 2\pi/a$ and a is the stripe structure period, the x-axis is directed along the surface normal to the stripes and the magnetization is directed normal to the surface. This model has the following advantages: (i) it is very simple; (ii) it correctly reproduces the main features of the periodic magnetic field created by periodic magnetization distributions. Consider this system in an external homogeneous magnetic field $\vec{B}_0 = (B_{0x}, B_{0z})$. The magnetic field distribution can be found by using the electrostatic analogy and solving Laplace's equation with the surface charge density $M = M_0 \sin(kx)$ at $z = 0$. The magnetic field distribution has the form ($z > 0$): $B_x = B_{0x} - 2\pi M_0 \cos(kx)\exp(-kz)$; $B_z = B_{0z} + 2\pi M_0 \sin(kx)\exp(-kz)$. The magnetic field energy density is given by ($z > 0$):

$$E_{lt} = \frac{1}{2} M_0 \exp(-kz)(\pi M_0 \exp(-kz) - B_{0x}\cos(kx) + B_{0z}\sin(kx)) + E_0. \qquad (2)$$

where $E_0 = (1/8\pi)(B_{0x}^2 + B_{0z}^2)$. The energy density changes periodically with x. It is straightforward to check that the potential in Eq.2 provides a set of stable energy minima which is periodic in the x direction. The position of the energy minima in Eq.(2) can be changed continuously by changing the external magnetic field $\vec{B}_0$, thus providing a method to transport particles along the surface. It is straightforward to show that the potential of Eq.2 can provide trapping of diamagnetic rods. The stable trapping of flat objects like membranes can be also realized in the case when the size of this object is commensurate with the magnetic structure period a. In the case of the infinite membrane, the force in the z direction averages and provides the net repulsive force. This repulsive forcenyura79

per unit area of the infinite plate with diamagnetic susceptibility $\chi < 0$ and thickness d is given by $F = 4\pi^2|\chi|M_0^2 kd\exp(-2kz)$. The detailed analysis of the potential (Eq.2) will be published elsewhere.

2-D Array of Magnetic Traps: As simple model of the two-dimensional array of magnetic traps we propose to use a magnetization distribution of the form $M = M_0 \sin(kx)\sin(ky)$, where $k = 2\pi/a$ and a is the structure period and the magnetization is directed normal to the surface in the z-direction. This model has the same advantages as the model for an array of the linear magnetic traps. We consider the z component B_{z0} of the external field. The magnetic field distribution has the form: $B_x = -\sqrt{2}\pi M_0 \cos(kx)\sin(ky)\exp(-\sqrt{2}kz)$, $B_y = -\sqrt{2}\pi M_0 \sin(kx)\cos(ky)\times$ $\exp(-\sqrt{2}kz)$ $B_z = B_{z0} + 2\pi M_0 \sin(kx)\sin(ky)\exp(-\sqrt{2}kz)$. The magnetic field energy density is given by ($z > 0$):

$$E_{st} = \frac{\pi}{4} M_0^2 (\exp(-2\sqrt{2}kz)(1 - \frac{1}{2}\cos(2kx) - \frac{1}{2}\cos(2ky)) + E_B \qquad (3)$$

where $E_B = (1/8\pi)B_{z0}^2 + (1/2)M_0 B_{z0}\sin(kx)\sin(ky)\exp(-\sqrt{2}kz)$. As in the case of potential of Eq.2 the potential of Eq.3 provides a periodic set of stable energy

minima which are bounded in the z-direction (the latter is due to the different z dependence of the first term and field dependent term E_B in Eq.3). Similar to Eq.2, the position of the energy minima in Eq.(3) can be changed continuously by changing the external magnetic field B_{0z}. As in the case of the potential of Eq.2, the stable trapping of flat objects like membranes can be realized for objects which are commensurate with the magnetic structure period a. In the case of the infinite membrane, the force in the z-direction averages and provides the net repulsive force. This repulsive force per unit area of the infinite plate with diamagnetic susceptibility $\chi < 0$ and thickness d_p is $F = \pi^2\sqrt{2}|\chi|M_0^2 k d_p \exp(-2\sqrt{2}kz)$. The detailed analysis of the potential in Eq.3 will be published elsewhere.

Rare-earth Magnetic Traps: Rare-earth magnets like samarium-cobalt and neodymium-iron-boron posses unique properties which can make possible stable levitation of diamagnetic materials on a submillimeter scale with commercially available magnets. The magnetic field on the surface of the rare-earth magnet is in the half a Tesla range. Commercially available magnets are fabricated by sintering 50-150 μm size powder. Assuming that such materials are still mechanically stable in the 100-200 μm range of thickness, it is feasible to have a typical magnetic field energy density gradient G_E of the order of $G_E \sim 10^4 \mathrm{T}^2/\mathrm{m}$. This value significantly exceeds the threshold[6] (1400 T^2/m) for water levitation. The best strategy is to implement linear arrays of magnetic traps. The layered structure is probably the simplest one for fabrication. Three possible configurations are: (1) layers of rare-earth magnets with alternating directions of magnetization; (2) layers of rare-earth magnets with parallel magnetization that are separated by intermediate layers of magnetically soft materials with a high saturation magnetization; (3) layers of magnetically soft material with a high saturation magnetization that a separated by intermediate non-magnetic layers with the whole assembly then placed in an external magnetic field. The properties of such systems are described above. In addition to layered structures the high coercive field of the rare-earth magnets can permit realization of some (or all) of the following geometries: (i) an array of magnetic columns with magnetization along long axes of the column in a checker-board-like alternating magnetization pattern; (ii) a thin magnetic plate with the magnetization normal to the plate but with a regular array of holes (anti-dots) drilled through the thickness of the plate by different methods (mechanically, electrochemically, by laser etc); (iii) an array of anti-dots on the face of the bulk rare-earth magnet with the magnetization normal to this face; (iv) an array of grooves in one or both directions cut on the face of the bulk rare-earth magnet with the magnetization normal to this face. To get the largest possible average G_E the total area of the anti-dots (grooves) should be equal to one-half of the face area. The list of possible array geometries can be easily enlarged to include more exotic magnetic superstructures, e.g. concentric rings of different depth, spherical or cylindrical profile of the surface of the bulk magnet, pie-like geometry with an alternating magnetization direction in each sector (piece), etc.

To conclude, I have shown that by using diamagnetism and nanofabrication it is feasible to trap and manipulate diamagnetic nanoparticles. Nanoscale magnetic

textures can provide stable static traps for nanosize diamagnetic systems like carbon nanotubes, proteins and membranes. Manipulation with trapped nanoparticles is possible via external magnetic fields. The captured nanoparticles are well isolated from the environment. The proposed traps can be also used in experiments with ultra-cold atoms and Bose-Einstein condensation.

I am grateful to D. G. Naugle, V. L. Pokrovsky and J. H. Ross for critical reading of the manuscript. This work was supported by NSF under the grant DMR 0103455 and by TAMU under the TITF grant.

References

1. S. Earnshaw, *On the nature of the molecular forces which regulate the constitution of the luminiferous ether*, Trans. Camb. Phil. Soc., 7, 97-112 (1839).
2. W. Thomson, *Reprint of Papers on Electrostatics and Magnetism* MacMillan, London, 1872, paper XXXIII, pp. 493-499, and paper XXXIV, pp. 514-515.
3. W. Braunbek, Z.Phys., **112**, 753-763 (1939); Z.Phys., **112**, 764-769 (1939);
4. A.Geim, Physics Today, Sep.1998, page 36-39
5. M.V.Berry and A.K.Geim, European Journal of Physics, v. 18, p. 307-313 (1997).
6. M.D.Simon, L.O.Heflinger and A.K.Geim, Am. J. Phys., **69**, pp. 702-713 (2001).
7. J. Reichel, W. Hänsel, and T. W. Hänsch, Phys. Rev. Lett. **83**, 3398 (1999).
8. W. Hänsel, J. Reichel, P. Hommelhoff, and T. W. Hänsch, Phys. Rev. Lett. **86**, 608 (2000).
9. W. Hänsel, P. Hommelhoff, T. W. Hänsch and J. Reichel, Nature **413**, 498 (2001)
9. J.D. Weinstein and K.G. Libbrecht, Phys. Rev. A, **52**, 4004-4009 (1995).
11. C.R. Martin, Chem. Mater., **8**, 1739 (1996).
12. Thurn-Albrecht T., Schotter J., Kastle C. A., Emley N., Shibauchi T., Krusin-Elbaum L., Guarini K., Black C. T., Tuominen M. T., Russell T. P., Science **290**, 2126-2129 (2000)
13. L. Sun, C. L. Chien and P. C. Searson, J. Mater. Sci. **35**, 1097 (2000).
14. R.L. Fleischer, P.B. Price, and R.M. Walker, *Nuclear Tracks in Solids,* University of California Press, Berkeley, 1975.
15. D. J. Sellmyer, M. Zheng, and R. Skomski, J. Phys.: Condens. Matter **13**, R433 (2001).
16 A. Despic and V.P. Parkhuitik, *Electrochemistry of aluminum in aqueous solutions and physics of its anodic oxide, Modern Aspects of Electrochemistry*, Plenum, NY vol. 20 (1989).